

Scene Understanding — Differentiable Graphics

Dr Fangcheng Zhong



Outline

- Inverse graphics
- Differentiable surface rendering
 - Differentiable rasterization
 - Physically-based differentiable rendering
- Non-surface representations
 - Light fields
 - Neural radiance fields
 - Differentiable volume rendering

Inverse Graphics



Rendering



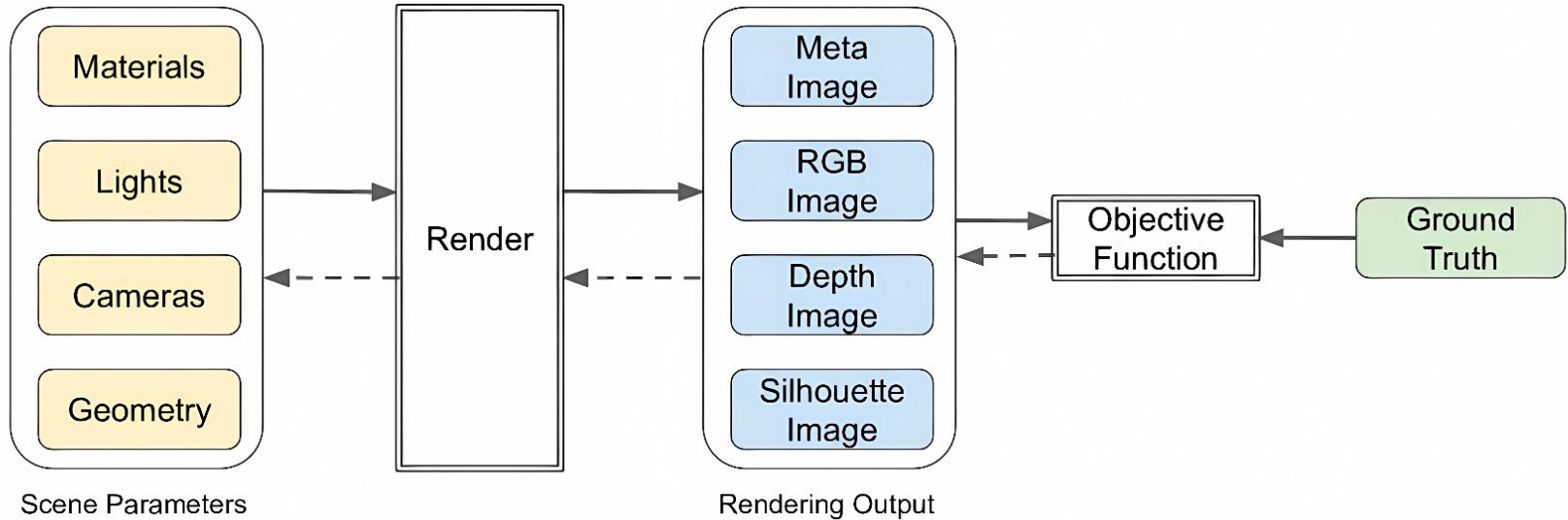
Inverse rendering



Inverse Graphics

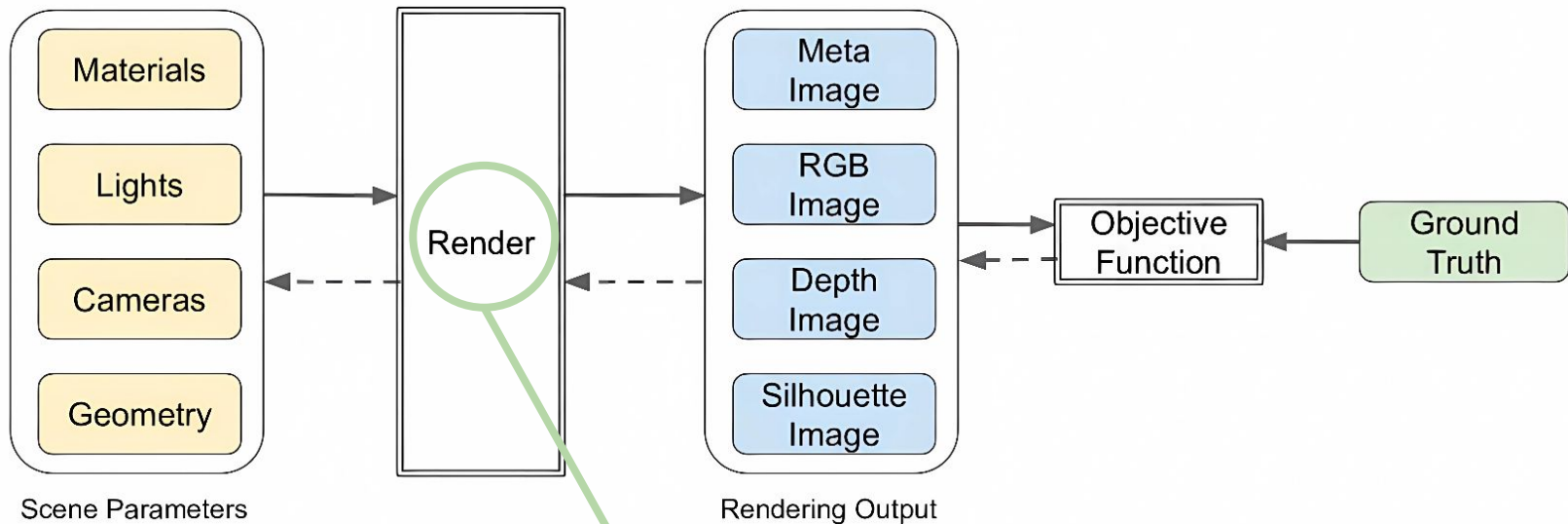
- Traditional approaches
 - SLAM, SfM
 - Light probes, structured light
- Data-driven approaches

Differentiable Rendering



generalised reprojection error minimisation

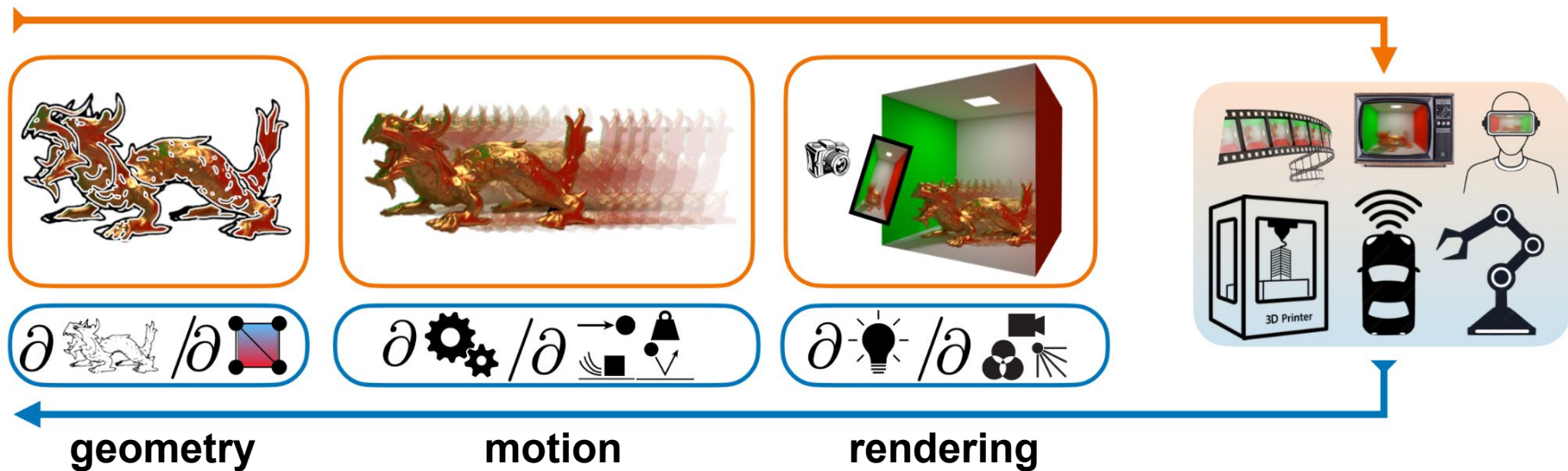
Differentiable Rendering



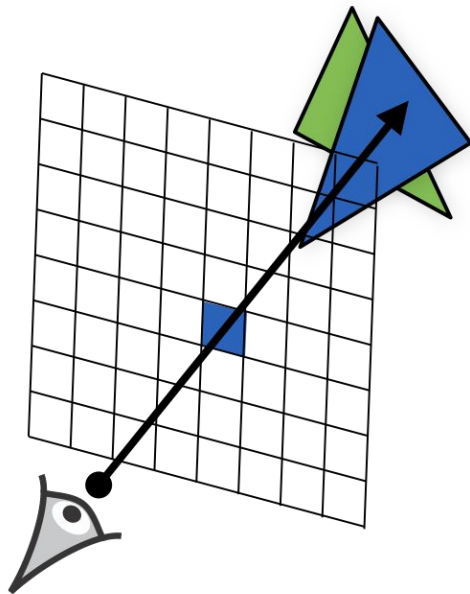
Derive useful gradients in rendering

Differentiable Graphics

Everything differentiable can be integrated!

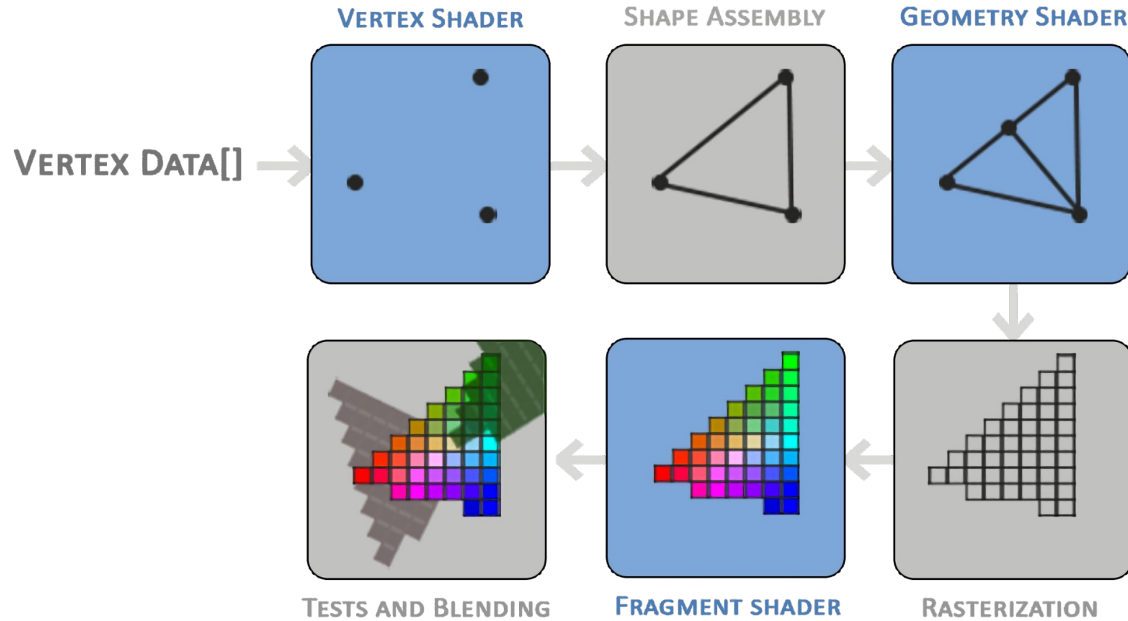


Differentiable Graphics



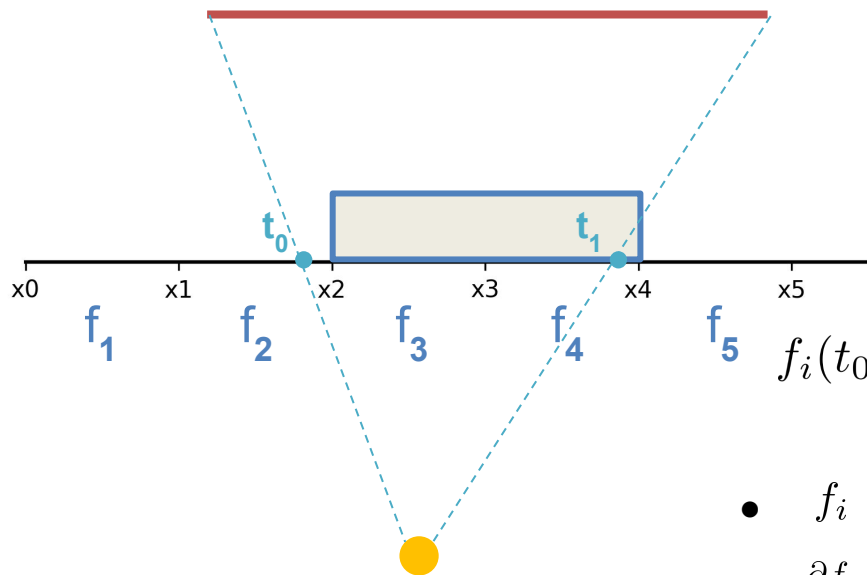
Visibility has no well-defined gradient

OpenGL Rendering Pipeline




Differentiable Rasterization

Rasterization has no gradient

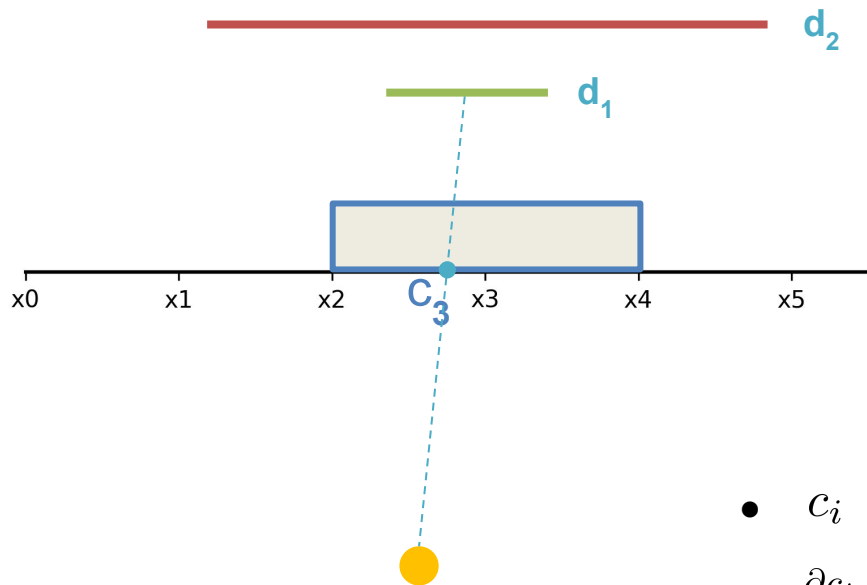


$$f_i(t_0, t_1) = \begin{cases} 1 & \text{if } [t_0, t_1] \cap [x_i, x_{i-1}] < \frac{x_i - x_{i-1}}{2} \\ 0 & \text{otherwise} \end{cases}$$

- f_i is the visibility of  at the i -th pixel location
- $\frac{\partial f_i}{\partial t_j}$ is zero or undefined everywhere

Differentiable Rasterization

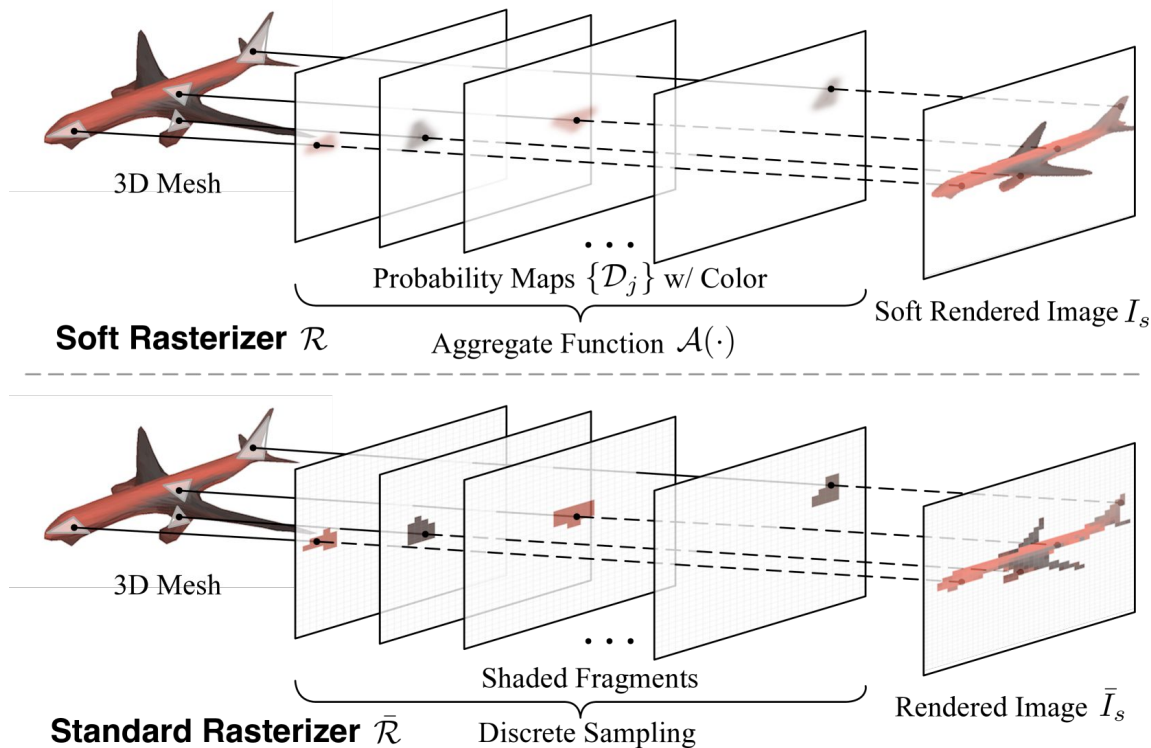
Z-buffering has no gradient



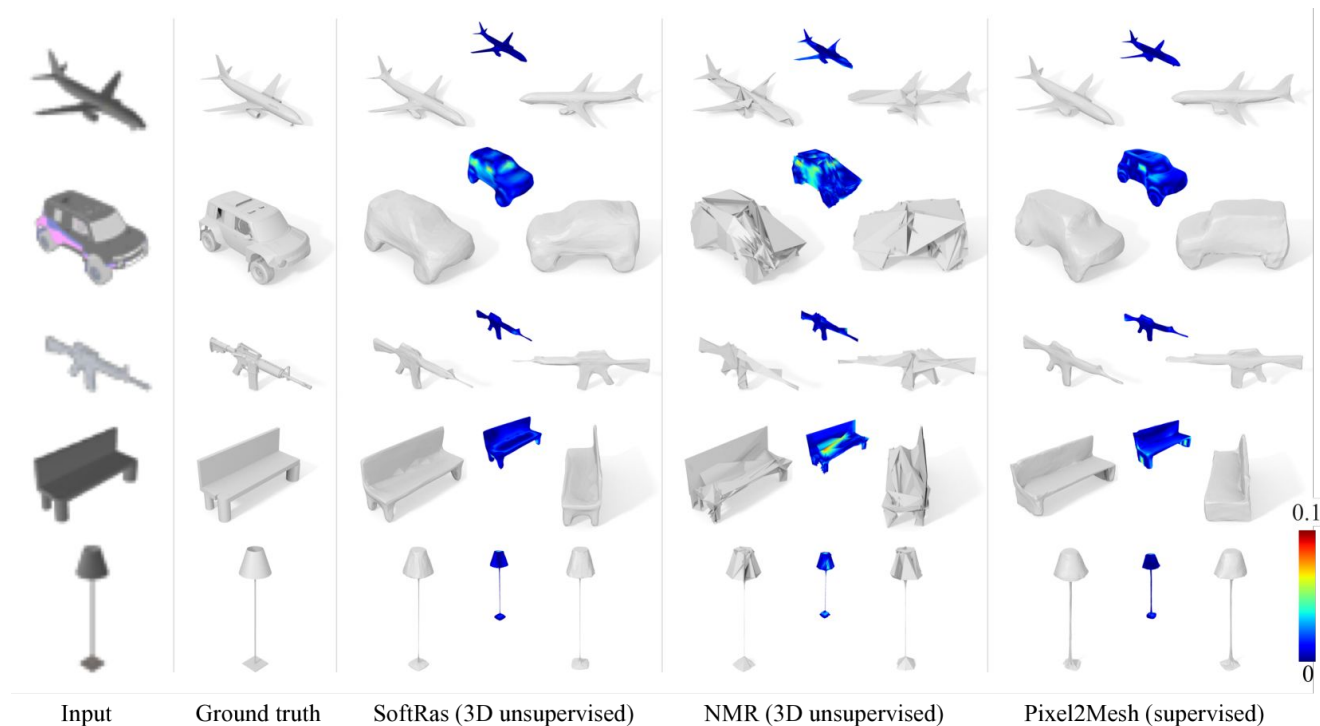
$$c_i(d_1, d_2) = \begin{cases} \text{green} & \text{if } d_1 < d_2 \\ \text{red} & \text{otherwise} \end{cases}$$

- c_i is the colour of i -th pixel location (if visible)
- $\frac{\partial c_i}{\partial d_j}$ is zero or undefined everywhere

Differentiable Rasterization



Differentiable Rasterization

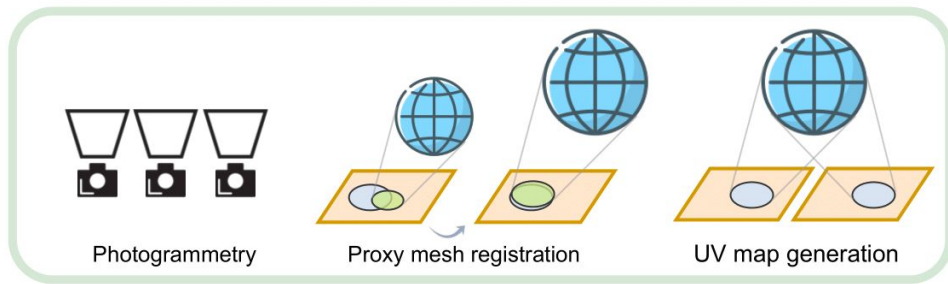


Differentiable Rasterization

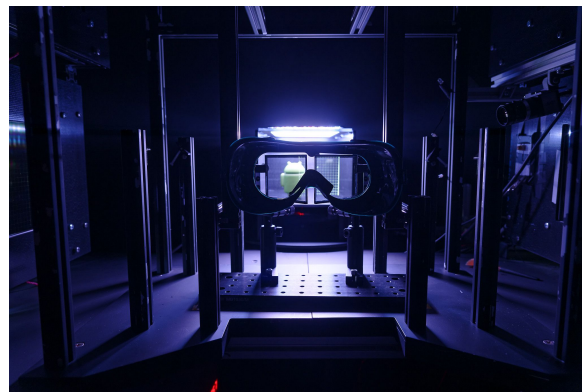
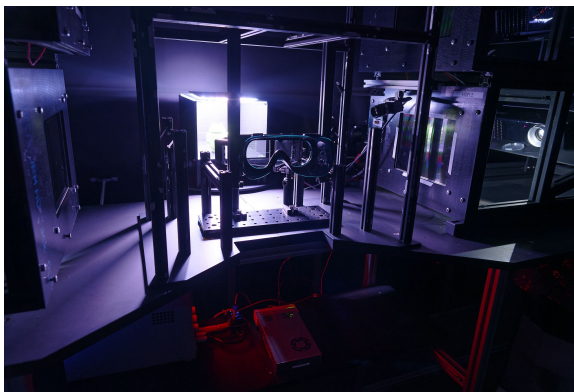
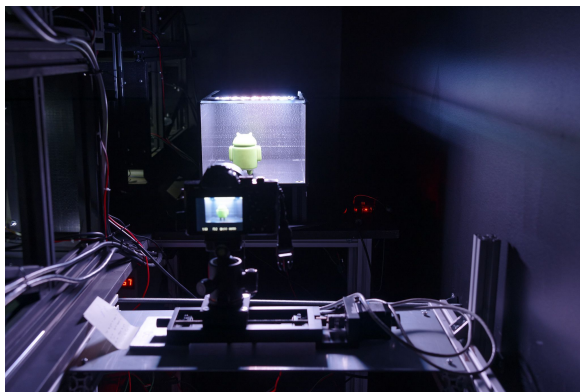
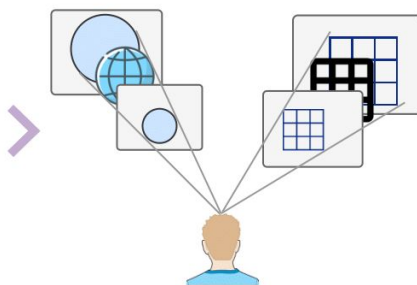
HDR light field capture



Lumigraph reconstruction



Focal plane calibration and rendering

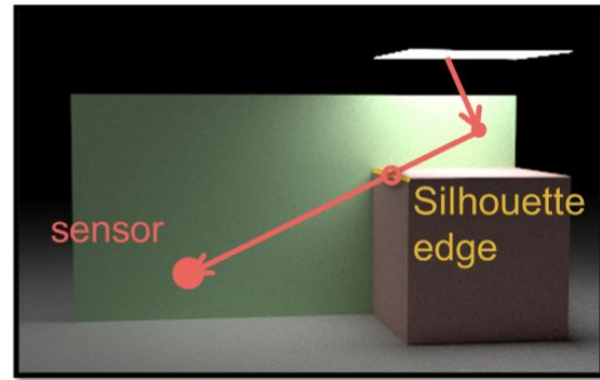
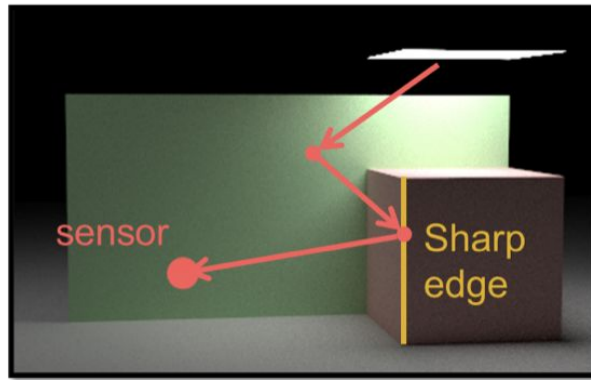
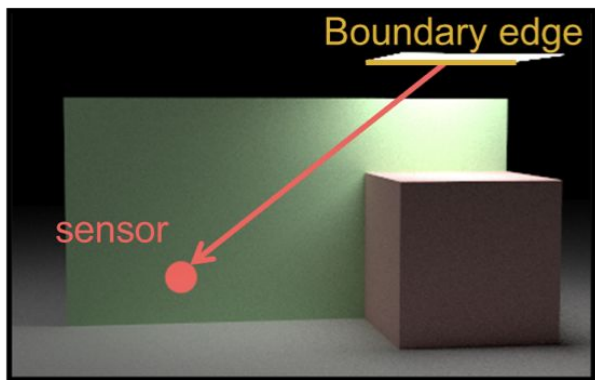


Physically-based Rendering

The rendering equation

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\Omega} f_r(\mathbf{x}, \omega_i, \omega_o) L_i(\mathbf{x}, \omega_i) \cos(\theta) d\omega_i$$

discontinuous!



Physically-based Differentiable Rendering

Differentiating the rendering equation

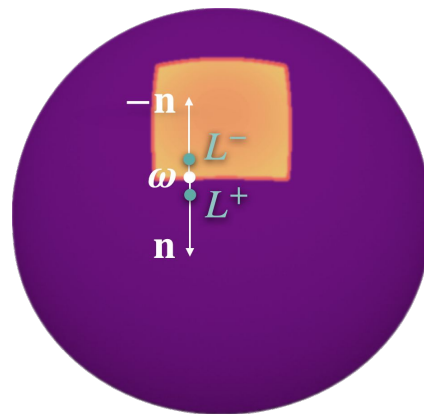
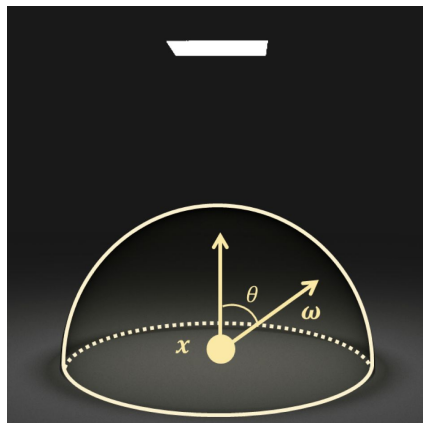
$$L_o(\mathbf{x}, \omega_o; \Theta) = L_e(\mathbf{x}, \omega_o; \Theta) + \int_{\Omega} f_r(\mathbf{x}, \omega_i, \omega_o; \Theta) L_i(\mathbf{x}, \omega_i; \Theta) \cos(\theta) d\omega_i$$

scene parameters

$$\frac{d}{d\Theta} L_o(\mathbf{x}, \omega_o; \Theta) \neq \frac{d}{d\Theta} L_e(\mathbf{x}, \omega_o; \Theta) + \int_{\Omega} \frac{d}{d\Theta} f_r(\mathbf{x}, \omega_i, \omega_o; \Theta) L_i(\mathbf{x}, \omega_i; \Theta) \cos(\theta) d\omega_i$$

Only true when the integrand is continuous

Physically-based Differentiable Rendering



boundary of the integration domain discontinuous boundary of L_i

$$\frac{d}{d\Theta} L_o = \frac{d}{d\Theta} L_e + \int_{\Omega} \frac{d}{d\Theta} f_r L_i \cos d\omega_i + \int_{\partial\Omega \cup \Omega^*} \mathbf{v} \cdot \mathbf{n} (L_i^- - L_i^+) f_r \cos dS$$

differential rendering equation

movement of S w.r.t. theta
in the normal direction

Physically-based Differentiable Rendering

Leibniz integral rule

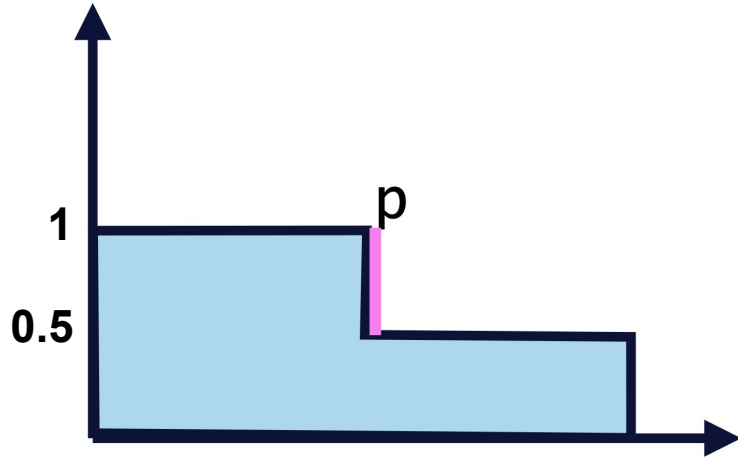
General form:

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(x, t) dt = f(x, b(x)) \frac{db}{dx} - f(x, a(x)) \frac{da}{dx} + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt$$

If *i*) $f(x, t)$ and its partial derivative $f_x(x, t)$ are continuous; *ii*) a and b are constant independent of x ,

$$\frac{d}{dx} \int_a^b f(x, t) dt = \int_a^b \frac{\partial}{\partial x} f(x, t) dt$$

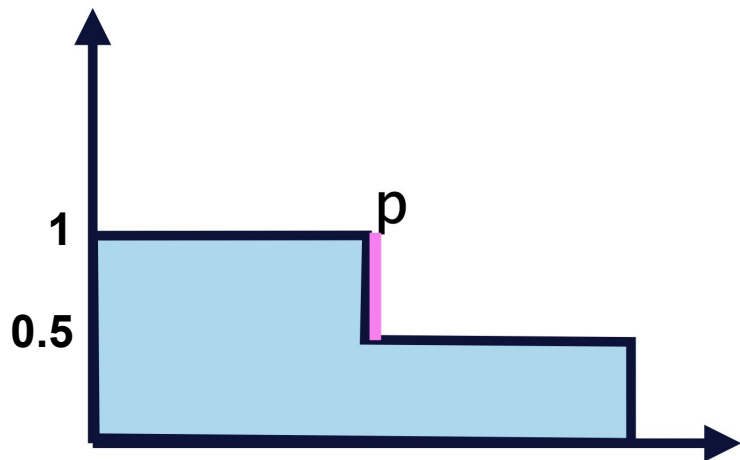
Physically-based Differentiable Rendering



$$\frac{d}{dp} \int_a^b t < p ? 1 : 0.5 dt$$

What if the integrand has sharp discontinuities (e.g. visibility) ?

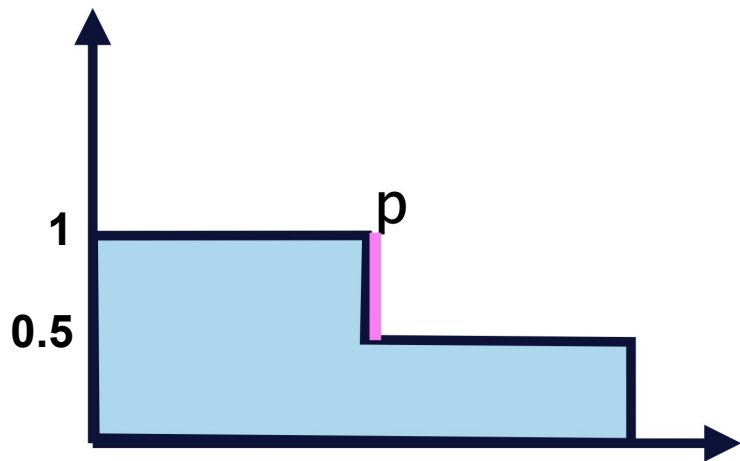
Physically-based Differentiable Rendering



$$\begin{aligned} & \frac{d}{dp} \int_a^b t < p ? 1 : 0.5 dt \\ &= \frac{d}{dp} \int_a^p 1 dt + \frac{d}{dp} \int_p^b 0.5 dt \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

Can apply the Leibniz rule to each term

Physically-based Differentiable Rendering



$$\begin{aligned}
 & \frac{d}{dp} \int_a^b t < p ? 1 : 0.5 dt \\
 &= \frac{d}{dp} \int_a^p 1 dt + \frac{d}{dp} \int_p^b 0.5 dt \\
 &= 1 - 0.5 \\
 &= 0.5
 \end{aligned}$$

$$\frac{\partial}{\partial p} \int \text{[Graph]} = \int \frac{\partial}{\partial p} \text{[Graph]} + \sum \text{[Graph]}$$

The diagram illustrates the differentiation of an integral with respect to a parameter p . The left side shows the derivative of the integral of a function. The right side is split into two parts: a light blue box containing the integral of the partial derivative of the function with respect to p , and a yellow box containing a summation of the difference between the function values on either side of the discontinuity, labeled $f_- - f_+$.

Physically-based Differentiable Rendering

Reynolds transport theorem

$$\frac{d}{dt} \int_{\Omega(t)} f(\mathbf{x}, t) dV(\mathbf{x}) = \int_{\Omega(t)} \frac{\partial}{\partial t} f(\mathbf{x}, t) dV + \int_{\partial\Omega(t)} f(\mathbf{x}, t) \mathbf{v} \cdot \mathbf{n} dA$$

generalisation of the Leibniz integral rule to higher dimensions

The diagram illustrates the Reynolds transport theorem. On the left, a blue and red arrow-shaped control volume is shown with a dashed black boundary. The expression is $\frac{\partial}{\partial p} \iint$. This is equal to the sum of two terms on the right. The first term is a volume integral $\iiint \frac{\partial}{\partial p}$ over the control volume, which contains two yellow dots. The second term is a surface integral \int over the boundary of the control volume, which contains four purple dots.

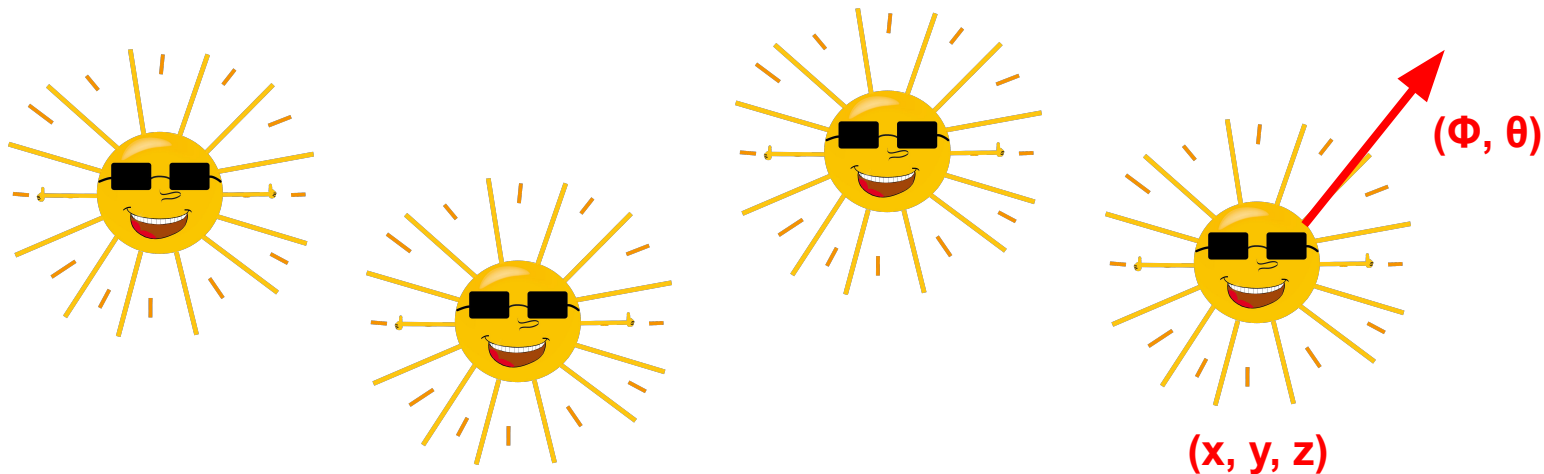
Physically-based Differentiable Rendering

Reparameterizing Discontinuous Integrands for Differentiable Rendering

Guillaume Loubet (EPFL) Nicolas Holzschuch (INRIA) Wenzel Jakob (EPFL)

SIGGRAPH Asia 2019

Light Fields



$$f(x, y, z, \phi, \theta)$$

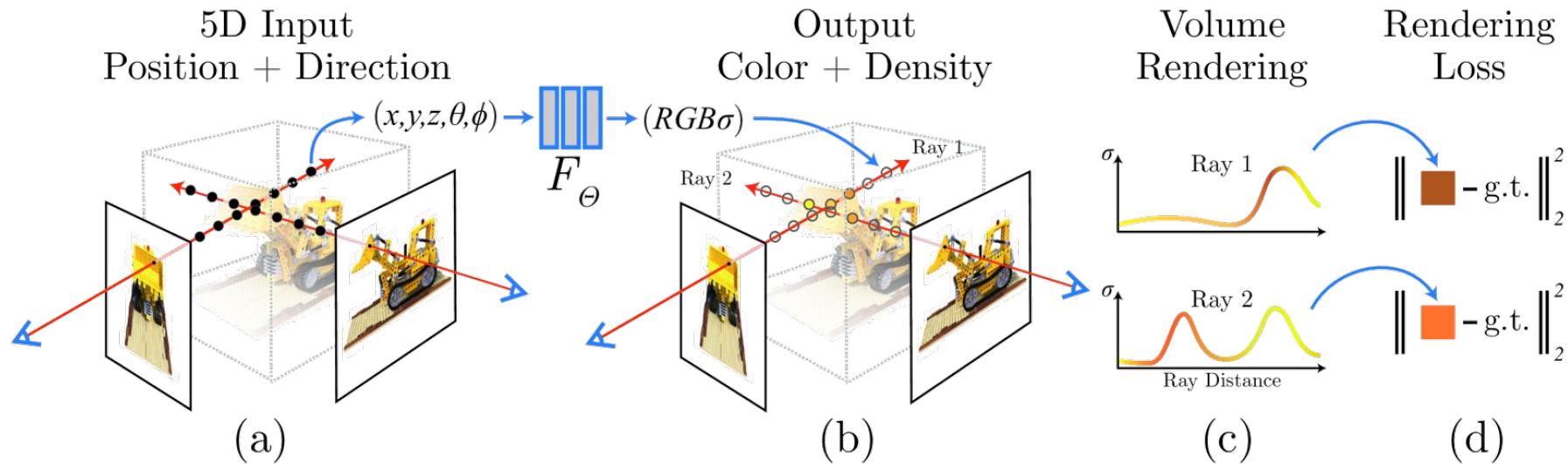
Incident radiance at an arbitrary location from an arbitrary direction

Light Fields

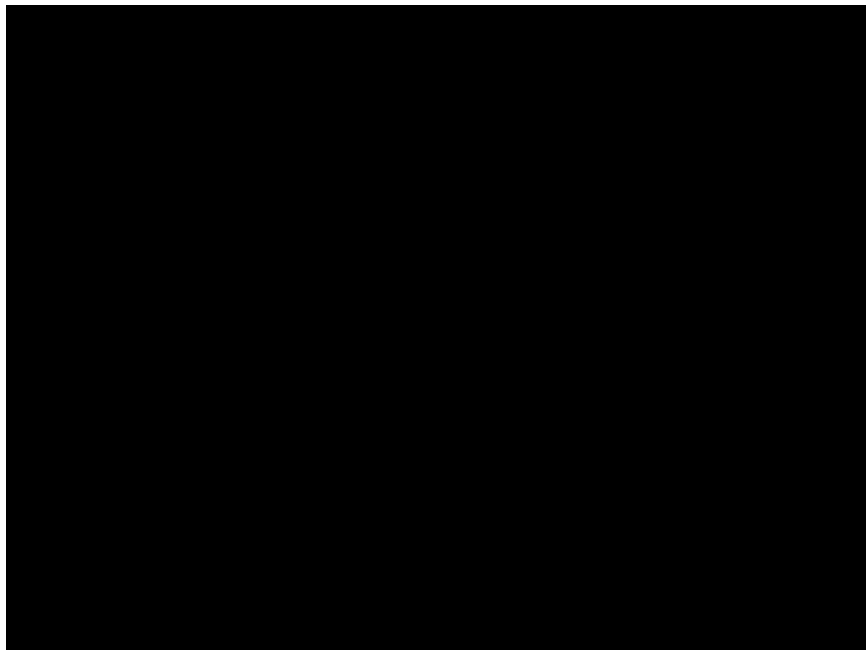


Broxton, Michael, et al. "Immersive light field video with a layered mesh representation." *ACM Transactions on Graphics (TOG)* 39.4 (2020): 86-1. <https://augmentedperception.github.io/deepviewvideo/>

Neural Radiance Field (NeRF)



Differentiable Volume Rendering




view synthesis

Differentiable Volume Rendering



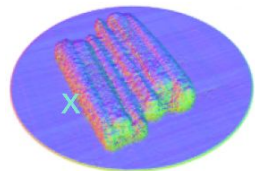
relighting

Differentiable Volume Rendering


$$= \int_{\mathcal{S}} \left(\text{(b) Light Visibility} \times \text{(c) Direct Illumination} + \text{(d) Indirect Illumination} \right) \times \text{(e) BRDF} d\omega_i$$

(b) Light Visibility (c) Direct Illumination (d) Indirect Illumination (e) BRDF

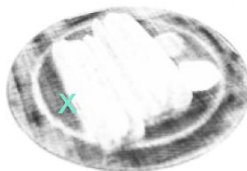
(a) Our Rendered Image
(Novel View and Lighting)



(f) Normals



(g) Albedo



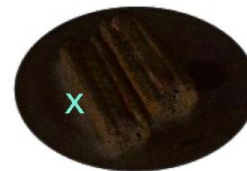
(h) Roughness



(i) Shadow Map



(j) Direct



(k) Indirect

Differentiable Volume Rendering



Differentiable Volume Rendering



Differentiable Graphics

- Unified framework to simultaneously infer multiple scene parameters
- Self-supervision
- Generalisability
- Cross regularisation
- Physics consistency in geometry and lighting